

Social Welfare Function of Economics (With Diagram)

Earlier neoclassical welfare theory, heir to the classical utilitarianism of Bentham, had not infrequently treated the Law of Diminishing Marginal Utility as implying interpersonally comparable utility, a necessary condition to achieve the goal of maximizing total utility of the society. Irrespective of such comparability, income or wealth *is* measurable, and it was commonly inferred that redistributing income from a rich person to a poor person tends to increase total utility (however measured) in the society.* But Lionel Robbins (1935, ch. VI) argued that how or how much utilities, as mental events, would have changed relative to each other is not measurable by any empirical test. Nor are they inferable from the shapes of standard indifference curves. Hence, the advantage of being able to dispense with interpersonal comparability of utility without abstaining from welfare theory.

- A practical qualification to this was any reduction in output from the transfer.

Auxiliary specifications enable comparison of different social states by each member of society in preference satisfaction. These help define Pareto efficiency, which holds if all alternatives have been exhausted to put at least one person in a more preferred position with no one put in a less preferred position. Bergson described an "economic welfare increase" (later called a *Pareto improvement*) as at least one individual moving to a more preferred position with everyone else indifferent. The social welfare function could then be specified in a *substantively* individualistic sense to derive Pareto efficiency (optimality). Paul Samuelson (2004, p. 26) notes that Bergson's function "could derive Pareto optimality conditions

as *necessary* but not sufficient for defining interpersonal normative equity." Still, Pareto efficiency could also characterize *one* dimension of a particular social welfare function with distribution of commodities among individuals characterizing *another* dimension. As Bergson noted, a welfare improvement from the social welfare function could come from the "position of some individuals" improving at the expense of others. That social welfare function could then be described as characterizing an equity dimension.

Samuelson (1947, p. 221) himself stressed the flexibility of the social welfare function to characterize *any* one ethical belief, Pareto-bound or not, consistent with:

- a complete and transitive ranking (an ethically "better", "worse", or "indifferent" ranking) of all social alternatives and
- one set out of an infinity of welfare indices and cardinal indicators to characterize the belief.

He also presented a lucid verbal and mathematical exposition of the social welfare function (1947, pp. 219–49) with minimal use of Lagrangean multipliers and without the difficult notation of differentials used by Bergson throughout. As Samuelson (1983, p. xxii) notes, Bergson clarified how production and consumption efficiency conditions are distinct from the interpersonal ethical values of the social welfare function.

Samuelson further sharpened that distinction by specifying the *Welfare function* and the *Possibility function* (1947, pp. 243–49). Each has as [arguments](#) the set of utility functions for everyone in the society. Each can (and commonly does) incorporate Pareto efficiency. The Possibility function also depends on technology and resource restraints. It is written in implicit form, reflecting the *feasible* locus of utility combinations imposed by the restraints and allowed by Pareto efficiency. At a given point on the Possibility function, if the utility of all but one person is determined, the remaining person's utility is determined. The Welfare function ranks different hypothetical *sets* of utility for everyone in the society from

ethically lowest on up (with ties permitted), that is, it makes interpersonal comparisons of utility. Welfare maximization then consists of maximizing the Welfare function subject to the Possibility function as a constraint. The same welfare maximization conditions emerge as in Bergson's analysis.

For a two-person society, there is a graphical depiction of such welfare maximization at the first figure of [Bergson–Samuelson social welfare functions](#). Relative to [consumer theory](#) for an *individual* as to two commodities consumed, there are the following parallels:

- The respective hypothetical utilities of the two persons in two-dimensional utility space is analogous to respective quantities of commodities for the two-dimensional commodity space of the indifference-curve *surface*
- The Welfare function is analogous to the indifference-curve *map*
- The Possibility function is analogous to the budget constraint
- Two-person welfare maximization at the tangency of the highest Welfare function curve on the Possibility function is analogous to tangency of the highest indifference curve on the budget constraint.

Arrow social welfare function (constitution)

[Kenneth Arrow \(1963\)](#) generalizes the analysis. Along earlier lines, his version of a social welfare function, also called a 'constitution', maps a set of individual orderings ([ordinal utility functions](#)) for everyone in the society to a social ordering, a rule for ranking alternative social states (say passing an enforceable law or not, [ceteris paribus](#)). Arrow finds that nothing of behavioral significance is lost by dropping the requirement of social orderings that are *real-valued* (and thus [cardinal](#)) in favor of orderings, which are merely *complete* and *transitive*, such as a standard [indifference curve](#) map. The earlier analysis mapped any set of individual orderings to *one* social ordering, whatever it was. This social

ordering selected the top-ranked *feasible* alternative from the economic environment as to resource constraints. Arrow proposed to examine mapping different sets of individual orderings to possibly different social orderings. Here the social ordering would depend on the set of individual orderings, rather than being *imposed* (invariant to them). Stunningly (relative to a course of theory from Adam Smith and Jeremy Bentham on), Arrow proved the general impossibility theorem which says that it is impossible to have a social welfare function that satisfies a certain set of "apparently reasonable" conditions.

Cardinal social welfare functions

A **cardinal social welfare function** is a function that takes as input numeric representations of individual utilities (also known as cardinal utility), and returns as output a numeric representation of the collective welfare. The underlying assumption is that individuals utilities can be put on a common scale and compared. Examples of such measures can be:

- life expectancy,
- per capita income.

For the purposes of this section, income is adopted as the measurement of utility.